in the following manner:

$$\nabla^2\begin{bmatrix} H_y \\ p \end{bmatrix} + [M]\begin{bmatrix} H_y \\ p \end{bmatrix} = [S] \tag{107}$$

where

$$[M] = \frac{1}{1-\beta}\begin{bmatrix} k_e^2 & \dfrac{-\omega\epsilon_0\epsilon_2}{N_0 e\epsilon_1} \dfrac{\omega^2\alpha\epsilon}{a^2\left(1-\dfrac{\omega_p^2}{\omega^2}\right)} \\ \dfrac{k_e^2\omega_p^2 B_0}{\omega^2-\omega_p^2} & k_a^2 \end{bmatrix} \tag{108}$$

and

$$[S] = -\frac{i\omega\epsilon_0\epsilon}{(1-\beta)\epsilon_1} J_0\delta(x)\delta(z)\begin{bmatrix} 1 \\ \dfrac{\omega_p^2 B_0}{\omega^2-\omega_p^2} \end{bmatrix}. \tag{109}$$

Introduce the following transformation:

$$\begin{bmatrix} H_y \\ p \end{bmatrix} = [T]\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}. \tag{110}$$

The substitution of (110) in (107) and the premultiplication by the inverse matrix $[T]^{-1}$ leads to

$$\nabla^2\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} + [T]^{-1}[M][T]\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = [T]^{-1}[S]. \tag{111}$$

If the matrix $[T]$ is chosen in such a way as to diagonalize $[M]$, the following two uncoupled wave equations are obtained:

$$\nabla^2\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} + \begin{bmatrix} k_{m0}^2 & 0 \\ 0 & k_{mp}^2 \end{bmatrix}\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = [T]^{-1}[S] \tag{112}$$

where k_{m0}^2 and k_{mp}^2, the eigenvalues of $[M]$, are given by the roots of the equation

$$(1-\beta)\lambda^2 - (k_a^2 + k_e^2)\lambda + k_a^2k_e^2 = 0. \tag{113}$$

From (113) it is clear that k_{m0}^2 and k_{mp}^2 are respectively the same as given in (38) and (39). The evaluation of the inverse matrix $[T]^{-1}$ yields the source term on the left-hand side of (112). Since the source terms are delta functions, the solutions are obviously Hankel functions.

ACKNOWLEDGMENT

The author wishes to thank Profs. R. W. P. King and T. T. Wu for help and encouragement with this research.

Radial-Line Coaxial Filters in the Microwave Region*

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Summary—**Design techniques and a simple empirical formula for the design of band rejection radial-line coaxial filters are presented. The appropriateness of these filters for parametric work is discussed and a particular structure employing these filters to provide a high performance harmonic filter structure for rectangular waveguide is presented.**

I. INTRODUCTION

SEVERAL requests for "further information" on radial-line coaxial filters followed the presentation of a paper[1] at the 1961 International Solid State Circuits Conference. This paper is a response to those requests and is intended to provide a practical design technique for the realization of these filters.

The design of coaxial filters in the microwave region above a few gigacycles has not received much attention in the past due to the popularity of rectangular waveguide for use at these frequencies. Coaxial filters in this frequency range have become increasingly important of late, however, due in large part to the advent of multiple frequency circuits employing coaxial lines (often in conjunction with other types of waveguides) which have come about through the application of solid-state art to microwave problems. Parametric amplifiers and frequency multipliers (or dividers) in particular have stringent filtering requirements for which coaxial filters of the type to be discussed in this paper seem particularly appropriate.

In addition, harmonic band rejection filters in rectangular waveguide structures are difficult to design for very good fundamental frequency performance and are often rather poor in their filtering response for one or more of the several harmonic waveguide modes that may be present. The problems associated with these filters can be avoided by accomplishing the filtering in a coaxial line and employing two rectangular wave-guide-to-coaxial line transducers.

* Received July 16, 1962; revised manuscript received September 26, 1962.
† Bell Telephone Laboratories, Inc., Holmdel, N. J.
[1] B. C. De Loach, Jr., "Waveguide parametric amplifiers," Digest of Technical Papers, 1961 Internat'l. Solid-State Circuits Conf., Lewis Winner, New York, N. Y.

II. Theory

We will limit our discussion to one particular type of band rejection coaxial filter known as a radial-line filter as depicted in Fig. 1. This filter is shown having a teflon dielectric radial-line section and air dielectric coaxial lines. Teflon dielectric coaxial lines have also been employed.

First we direct our attention to the choice of a and b. Perhaps the most stringent requirement on these parameters is that they not allow multimoding in the coaxial line in the range of frequencies to be rejected. It can be shown[2] that multimoding is possible for values of $(a+b)$ greater than or equal to that given by

$$a + b = \frac{K\lambda}{\pi} \quad \text{with} \quad 0.92 \leq K \leq 1.02 \tag{1}$$

with K a function of the impedance of the coaxial line.

By requiring that the loss of the coaxial line be minimized subject to (1) (where K is approximated by unity), we obtain[3] a ratio $b/a = 4.68$ which corresponds to a 92.6 Ω coaxial line if air dielectric is employed (see Appendix). A plot of $\alpha_c/\alpha_{c\ \min}$ vs b/a with $a+b$ held constant is presented in Fig. 2. α_c is that part of the coaxial line loss (in nepers per meter) that depends upon the choice of b/a. $\alpha_{c\ \min}$ is the value of α_c when $b/a = 4.68$. It can be seen that the minimum is a broad one and that from the minimum loss standpoint the choice of b/a is not critical.[4]

To summarize, a and b are determined from condition (1), *i.e.*, the sum $a+b$ is kept less than the value specified by (1), and from $b/a = 4.68$ in the absence of external requirements not herein considered.

We now wish to determine t and r for the radial line cavity of Fig. 1. Schelkunoff[5] has done theoretical work on the determination of the resonant frequency of cylindrical cavity resonators. He treats two specific cases of interest. The first, Fig. 3(a), is that of two coaxial conducting cylinders bounded by conducting planes perpendicular to their axes. The second, Fig. 3(b), is identical except that the inner conducting cylinder is replaced by an open circuit. Schelkunoff obtains their respective resonant frequencies from the equations

$$\frac{J_0(\beta b)}{N_0(\beta b)} = \frac{J_0(\beta r)}{N_0(\beta r)} \tag{2a}$$

$$\frac{J_1(\beta b)}{N_1(\beta b)} = \frac{J_0(\beta r)}{N_0(\beta r)} \tag{2b}$$

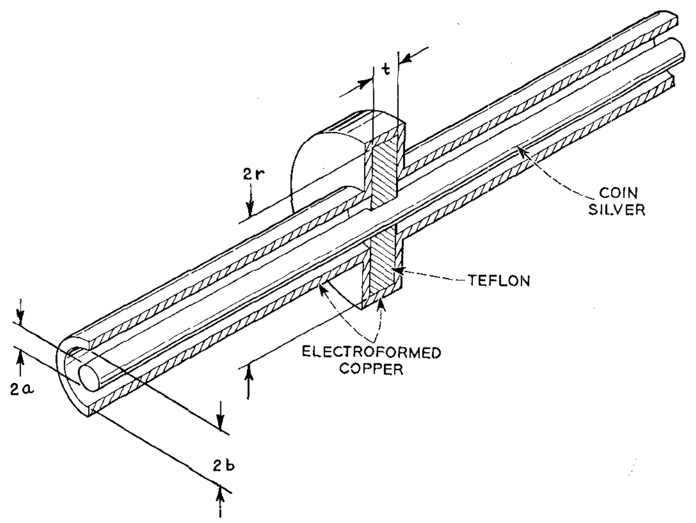

Fig. 1—Radial-line coaxial filter (cutaway).

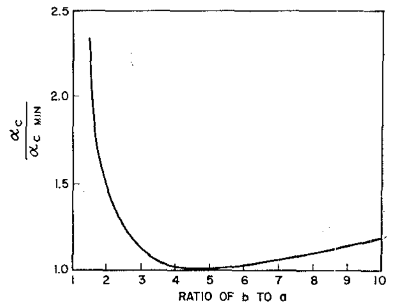

Fig. 2—Dependence of loss on the ratio of b to a.

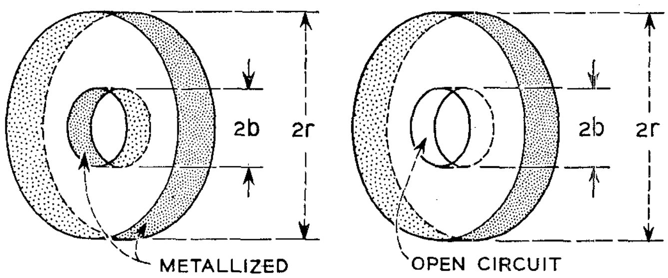

Fig. 3—Two configurations treated by Schelkunoff.

Obviously the radial-line coaxial filter that we are treating fits neither of these cases exactly; however, by making an arbitrary choice of $t = \lambda_{air}/10$ for all our filters, with λ_{air} the wavelength to be rejected, we find that (2b) more closely approximates our experimental observations than does (2a).

From an observation of the experimental values presented in this paper, and from others (which for brevity have been deleted), the relation

$$2r = (a + b) + \frac{\lambda_{air}}{2} \tag{3}$$

has been found to closely approximate the correct value of r for given a, b, and λ_{air}, under the conditions

[2] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," McGraw-Hill Book Co., Inc., New York, N. Y., p. 42; 1948.

[3] G. L. Ragan, "Microwave Transmission Circuits," McGraw-Hill, Book Co., Inc., New York, N. Y., pp. 146–147; 1948.

[4] The common minimum loss ratio[3] of 3.6 based on the fixing of the outer conductor diameter does not apply here and in fact, would yield 1.02 times as much loss in nepers per meter as the 4.68 ratio.

[5] S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Co., Inc., New York, N. Y., ch. 8; 1943.

$t=\lambda_{air}/10$, and the use of teflon in the radial-line section as shown in Fig. 1. Its simplicity makes it an attractive alternate to (2b) and its application and accuracy will be discussed in Section III.

III. Experiment

Our experimental realization of these radial-line filters is accomplished as follows:

1) a and b are chosen from the requirements that $b/a=4.68$, and that $a+b=C_1$. C_1 is a constant and is a fractional part of the sum $a+b$ given by (1) (see Appendix).
2) t is determined by $t=\lambda_{air}/10$ for the frequency to be rejected.
3) Teflon dielectric is employed in the radial line section and either air or teflon dielectric is chosen for the coaxial line.
4) A theoretical estimate is made for r from (3) (or from (2b)).
5) The correct value of r is determined experimentally by constructing filters employing a scatter of values of r about the theoretical prediction.

To examine the effect of our choice of $t=\lambda_{air}/10$ upon the resonant frequency of these filters, a set was constructed (electroformed), all of which had $2a=0.031$ in, $2b=0.640$ in, teflon dielectric radial line and coaxial lines, but different values of t. The results are presented in Fig. 4.

The limitations inherent in the application of Schelkunoff's equation (2b) are obvious in that it is independent of t while the resonant frequency obviously is not. The predicted point of our empirical relation is shown for $t=\lambda_{air}/10=0.106$ in. It is in error by approximately 1.5 per cent. It is seen that (3) should give good results as long as t is reasonably close to $\lambda_{air}/10$. The following three experiments were performed to try to determine the extent of the applicability of (3).

Experiment One: A filter was electroformed which had $2a=0.031$ in, $2b=0.193$ in, $t=0.125$ in, $2r=0.645$ in, teflon dielectric coaxial line and radial lines. The hole in the teflon dielectric was then enlarged to accommodate a center conductor of first 0.062-in diameter and then 0.125-in diameter. The resonant frequency was determined in each case and the results are presented in Fig. 5. The linear relation expressed by (3) is included and is seen to be a reasonably good approximation, and to be in error by approximately 2.0 per cent at the highest frequency measured. It should perhaps be emphasized that Schelkunoff's equation (2b) is independent of a and would predict a resonant frequency of 11.08 Gc for all these filters.

Experiment Two: To examine the effect of the radius of the outer conductor of the coaxial line on the resonant frequency, a series of filters was fabricated, all of which had $2a=0.031$ in, $2r=0.640$ in, $t=0.125$ in, teflon dielectric coaxial line and radial line, but with different values of b. The results are presented in Fig. 6. Eqs. (2b) and (3) are also plotted for this case. Eq. (3) is seen to yield excellent results except for values of b very near to a.

Experiment Three: Three sets of coaxial filters were fabricated which had the following properties:

Set	$2a$	$2b$	t	Coaxial-line dielectric	Radial-line dielectric	Frequency for which $t=\lambda_{air}/10$
1	0.031 in	0.193 in	0.125 in	teflon	teflon	9.44 Gc
2	0.131 in	0.302 in	0.125 in	air	teflon	9.44 Gc
3	0.050 in	0.234 in	0.062 in	air	teflon	19.0 Gc

but which had different values of r. All three sets of filters will be observed to satisfy condition (1), *i.e.*, the sum $a+b$ is less than that specified by Eq. 1. while the third set in addition employs a minimum loss coaxial line. The results are presented in Figs. 7–9 and include plots of (2b) and (3). Strictly speaking, (3) is to be employed only at $t=\lambda_{air}/10$, but as is obvious from Fig. 4, it should be a reasonably good estimate in the vicinity of $\lambda_{air}/10$ and is indeed observed to be so in Figs. 7–9, as it was in Figs. 5 and 6.

Removal of the teflon dielectric from the coaxial lines of the filters of Set 1 lowered each resonant frequency by approximately 70 Mc. Schelkunoff's relation (2b) is seen to give excellent results in Sets 1 and 2, but poor results for Set 3.

Radial-line coaxial filters have very "sharp" band rejecting properties. If, however, the rejection bandwidth is too narrow, one can greatly enhance it through the simple expedient of "stacking" two or more filters in series along a coaxial line. The responses of one-, two-, and three-section filters of this type are presented in Fig. 10. Inappropriate spacing can deteriorate the band rejection properties remarkably. The transmission characteristics for a range of spacings of a two-section filter are presented in Fig. 11. If the reflecting plane of a given filter section (at the resonant frequency) were at the center of the section, we would expect $n\lambda/2$ ($n=1, 2, 3, \cdots$) spacing to be appropriate. This, however, is not the case, and indeed for the filters of Fig. 11, $(2n+1)\lambda/4$ ($n=0, 1, 2, \cdots$) is a much better estimate of the correct spacing. The variation of the position of the reflecting plane with the dimensions of the filter section can only be very crudely estimated at present, and it is felt that the correct spacing for a given filter design should be determined experimentally with the $(2n+1)\lambda/4$ spacing a good first estimate in the majority of cases.

The sharpness of the band rejection characteristic has one interesting and useful consequence. At half the

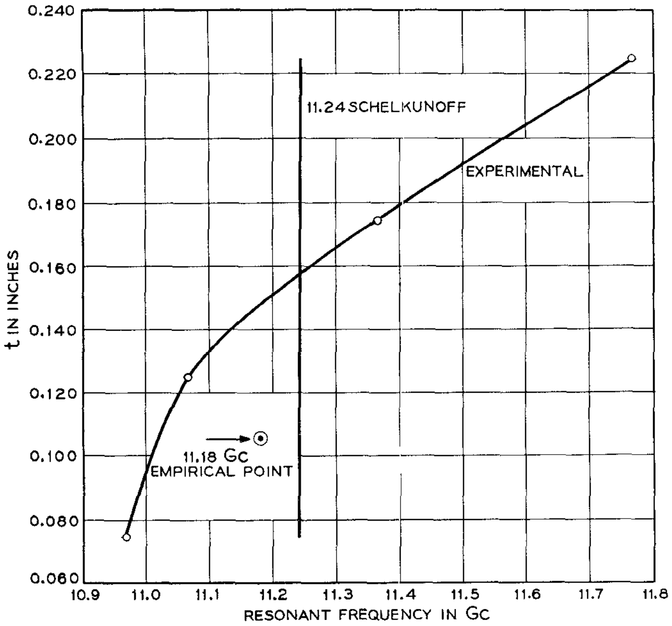

Fig. 4—Dependence of resonant frequency of radial-line filter on *t*.

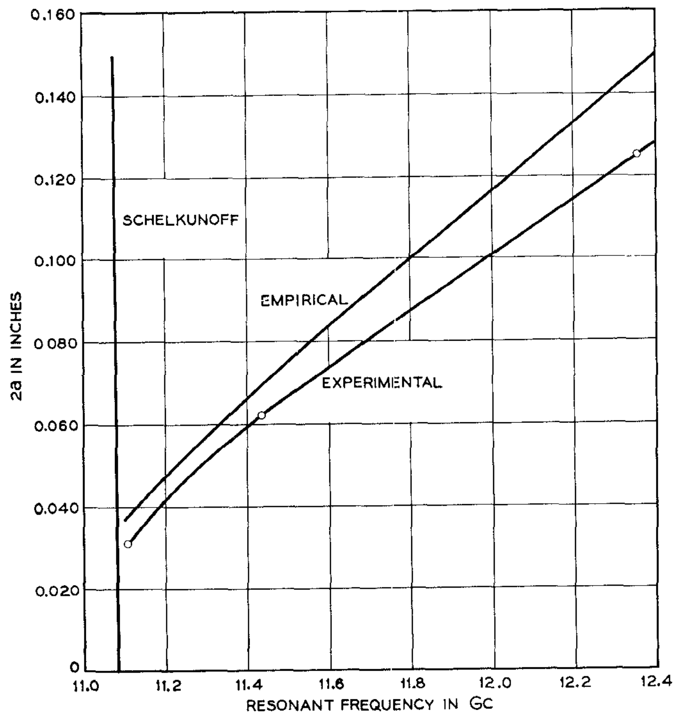

Fig. 5—Dependence of resonant frequency of radial-line filter on *a*.

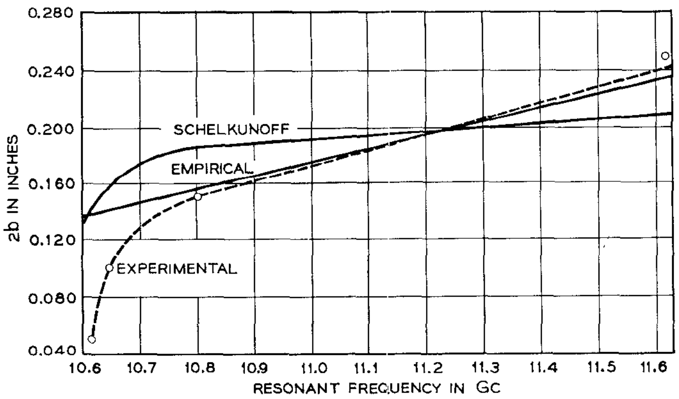

Fig. 6—Dependence of resonant frequency of radial-line filter on *b*.

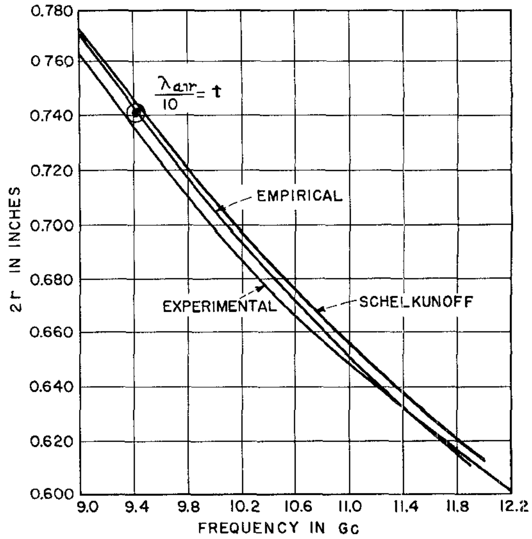

Fig. 7—Resonant frequency vs 2r for Set 1.

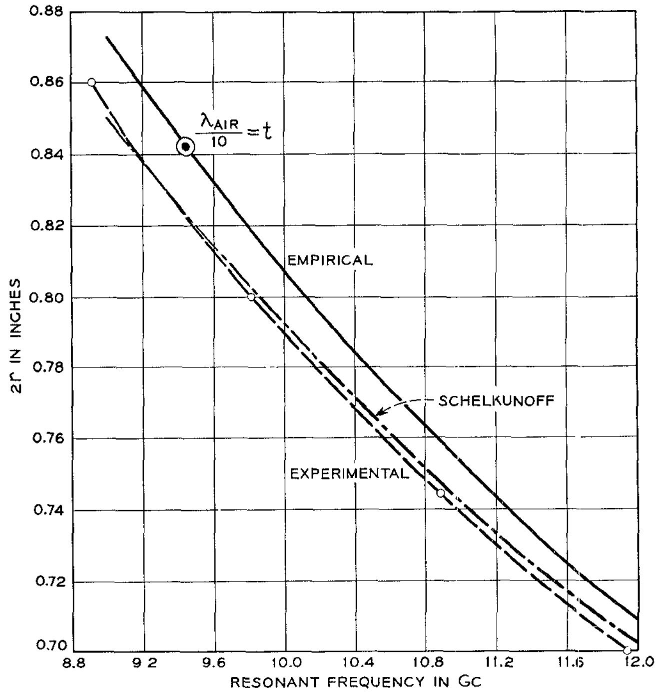

Fig. 8—Resonant frequency vs 2r for Set 2.

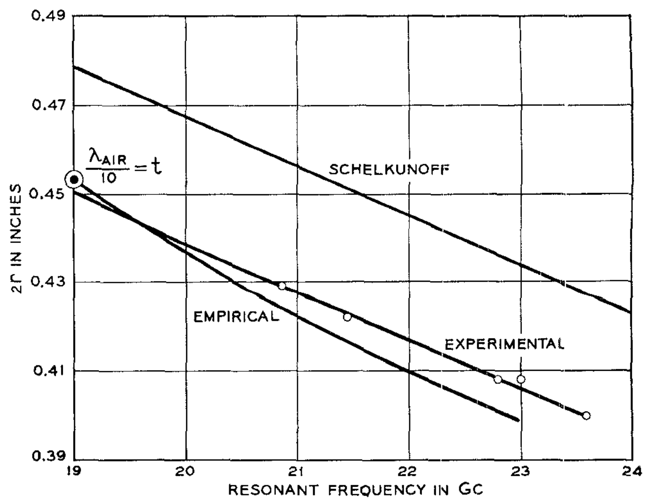

Fig. 9—Resonant frequency vs 2r for Set 3.

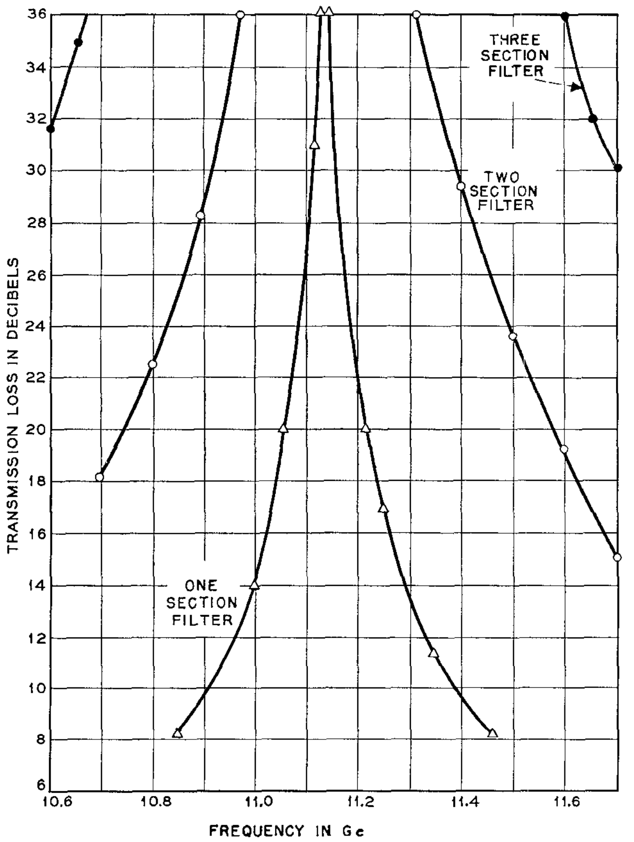

Fig. 10—Transmission characteristics of one- two-, and three-section radial-line coaxial filters.

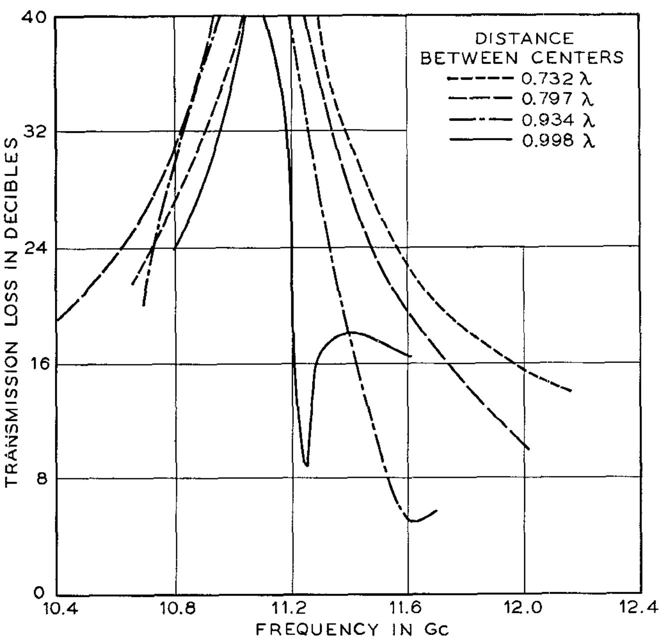

Fig. 11—Transmission characteristics of a two-section filter as a function of the spacing between centers.

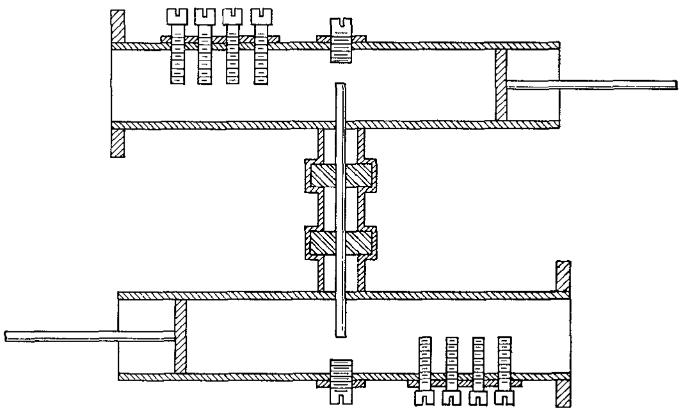

Fig. 12—Waveguide harmonic filter utilizing radial-line filter and two transducers.

band rejection frequency they are almost reflectionless with return losses of those measured (of Type 3) in excess of 30 db. This makes filters of this type ideally suited for multiple frequency circuits which are commonly encountered in parametric amplifier and mixer work.

One particular embodiment of a harmonic filter for use in such circuits was constructed as follows:

Mumford[6] has given an experimental procedure which was followed to broad-band a transition from 0.400 in $\times 0.900$ in waveguide to a coaxial line with $2a=0.050$ in, $2b=0.234$ in and with air dielectric. A two-section coaxial filter with teflon dielectric radial line and air dielectric coaxial lines with these dimensions, designed to band reject at 23.1 Gc, (see Fig. 9), was inserted between two of these transducers with trimmer tuning screws provided as shown in Fig. 12. The X-band return loss of this combination was greater than 40 db from 10.7 to 11.7 Gc, and the transmission loss over this range was 0.1 db. The K-band transmission loss was greater than 60 db over a 1-Gc band centered around 23.1 Gc. This is better than one might expect from extrapolation of the curves of Fig. 10 and is probably due to the fact that considerable mismatch occurs in the coaxial-to-waveguide transducers for the band rejection frequencies.

IV. Discussion

The experimental work presented had as its objective appropriate design criteria for radial-line coaxial band rejection filters. The effects of the different parameters upon the resonant frequency of these filters were investigated, and a particularly simple empirical formula was presented for radial-line coaxial filters employing teflon dielectric.

The full range of application of this formula is not known although it has been singularly successful in giving approximate values of filter diameter 2r under

[6] W. W. Mumford, "The optimum piston position for wide-band coaxial-to-waveguide transducers," Proc. IRE, vol. 41, pp. 256–261; February, 1953.

the conditions specified in our experimental procedure.

The application of Schelkunoff's work which has been used in previous radial-line filter design[7] was investigated.

Attention was called to the fact that radial-line coaxial filters have a high return loss at frequencies well below their resonant frequency which makes them particularly appropriate for circuits commonly occurring in parametric work.

The waveguide harmonic filter shown in Fig. 11 has been utilized in parametric amplifier work[1] and due to its excellent signal performance has solved one of the filtering problems associated with waveguide parametric amplifiers.

Appendix

The derivation of the minimum loss ratio in Ragan[3] approximates relation (1) by

$$\lambda = \pi(a + b)$$

and then minimizes the coaxial-line loss subject to this relation. It is perhaps comforting to know that if a designer computes the multimoding limit of the sum $a+b$ from relation (1) and then picks some percentage of this as a practical design limit (*i.e.*, he will not work exactly at the point of multimoding) that the condition $(a+b)$ equal to a constant, when combined with the minimum loss requirement, leads to exactly the same ratio. Thus the minimum loss ratio becomes of practical import.

[7] "The Microwave Engineers Handbook," Horizon House—Microwave Inc., Brookline, Mass., pp. TD-49; 1961–62.

Acknowledgment

Thanks are due S. F. Jankowski who did the electroforming and many of the measurements. The author also wishes to express thanks to C. F. Edwards of the Holmdel laboratory for many informative discussions.

Periodic Cylinder Arrays as Transmission Lines*

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Summary—**Periodic structures of conducting cylinders have been used as radiators (Yagi antennas), and, more recently, as slow-wave lines in traveling-wave tubes and masers. In this report it is shown that a nonresonant structure may have interesting capabilities as an open surface-wave transmission line. By means of a relatively simple matching network, efficient excitation of a surface wave on the periodic line is obtained. Response is flat over a 20 per cent frequency range at *X* band for several combinations of cylinder lengths and spacings. Total insertion losses are less than 3 db and largely independent of length of transmission line. Conducting cylinders are embedded in styrofoam.**

The effects of bends and twists in the line have also been investigated. It is shown experimentally that a guided wave on this periodic structure can follow a circular path having 1.5λ radius of curvature with very little loss. The plane of polarization can be rotated 90° by inserting a short twisted section.

By terminating the transmission line with short circuits at both ends, a discrete series of transmission maxima is observed. Since these resonant peaks of transmission are of high *Q* factor, the dispersion characteristic of the line is obtained with very good accuracy.

This type of open transmission line may offer advantages over heavy-weight and bulky conventional waveguides for some specialized applications.

I. Introduction

A NUMBER OF infinitely long periodic structures theoretically support a propagating plane wave along their axes.[1,2,3] In practice, the guiding structures are of finite length, and any desired propagating mode has to be excited by a set of currents which can neither be infinite in amplitude nor can they be distributed over an infinite aperture in space. Another complication arises from the fact that the structure is terminated and we have also the reflected wave to consider. For these reasons, the performance one obtains experimentally is often substantially different from theoretical predictions. However, quite useful approximations may be obtained, and in many cases one is in a position to estimate the bounds of the error.

As a first step in calculating propagation characteristics and excitation efficiency, it is essential to know the field configuration of the wanted mode of propagation. These fields must be a solution to Maxwell's equations and must satisfy boundary conditions at the guiding interface. At the exciting end these fields must, at least approximately, match the fields impressed by the launching device. For many periodic structures, it is relatively straightforward to formulate the total field

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[1] F. D. Borgnis and C. H. Papas, "Electromagnetic waveguides," in "Encyclopedia of Physics," vol. 16, Springer-Verlag, Berlin, Germany,; 1958.

[2] L. Brillouin, "Wave guides for slow waves," *J. Appl. Phys.*, vol. 19, pp. 1023–1041; 1948.

[3] F. J. Zucker, "The guiding and radiation of surface waves," *Proc. Symp. on Modern Advances in Microwaves Techniques*, Polytechnic Institute of Brooklyn, N. Y.; 1954.